

# The understanding of numerical order and its role in the development of arithmetic abilities

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*The development of mathematical abilities constitutes a crucial  
foundation for our modern and educated societies.*

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## Executive summary

- *Numerical order* refers to the relative rank or position of a numeral within a sequence.
- Knowledge about numerical order constitutes an important numerical dimension that can be differentiated from knowledge about numerical magnitude.
- Individual differences in the ability to process numerical order are associated with arithmetic abilities. This relationship increases over the first years of formal education.
- Specific brain regions in the left parietal cortex respond to numerical order processing. The brain activity of this region also increases with age—probably reflecting a more automatized and efficient processing of numerical order.
- The relationship between numerical order processing and arithmetic abilities might also be mediated by brain regions that are involved with controlling the flow of semantic information.
- Children with developmental dyscalculia show difficulties and different brain responses in relation to numerical order processing compared to their typical peers.
- Fostering the development of mathematical abilities encompasses the integration of multiple aspects associated with symbolic number knowledge. These range from knowledge about numerical order to knowledge about numerical magnitudes.
- Teaching methods that highlight the structure of numerical dimensions are powerful tools to foster numeracy in children. Children, especially those coming from families with a low socioeconomic status, often lack the opportunities to interact with material that displays a structured symbolic numerical content.

## Introduction

The development of mathematical abilities constitutes a crucial foundation for our modern and educated societies. Research has shown that mathematical abilities are as important for life success as literacy<sup>[1]</sup> and that deficits in this domain can have severe effects on an individual's well-being and on a nation's economy<sup>[2]</sup>. Current estimates show that around 20% of the population in countries of the Organization for Economic Co-operation and Development (OECD) have difficulties with mathematics, imposing great practical and occupational restrictions<sup>[3]</sup>. These numbers demonstrate the need to better understand the development of mathematical abilities. Knowledge about how the child's brain learns the core concept of mathematics is crucial in order to optimally foster its development.

The last decades of *neurocognitive* research have shown that the development of mathematical abilities is tightly linked to the mental ability to represent the meaning of number symbols. Current endeavors to better understand this mental representation focus on questions like: What are the crucial numerical dimensions that number symbols (such as Arabic digits and number words) represent? Where and how does the human brain represent these dimensions? And how do they relate to the development of mathematical abilities, specifically to the acquisition of arithmetic skills? In the past years, two important numerical dimensions have been identified to play a central role: **numerical magnitude** and numerical order. The present article focuses on the principles of numerical order, since numerical magnitude is covered in another article of this website (see "[How does the child's brain process numerical magnitude? Implications for learning mathematics](#)").

### What is numerical order?

It is well-known that numbers convey different meaning in different contexts (see Figure 1). For instance, as adults we can shift quite effortlessly between knowing that the Arabic numeral "3" refers to three apples, or, that the numeral "3" refers to the third runner in a race. The first example, three apples, refers to the notion that Arabic numerals express numerical magnitudes. The second example, refers to another numerical dimension which has gained much interest within the last years—numerical order. Numerical order can be defined as the knowledge about the relative rank or position of a numeral within a sequence<sup>[4-6]</sup>. Knowledge about the ordinal position of numbers allows us, for example, to infer with relative ease that one-thousand-and-two comes right after one-thousand-and-one. This efficient judgment would be impossible if numerals could only convey numerical magnitudes. Because of its obvious relevance, the number of neurocognitive studies

investigating how children and adults process numerical order has significantly increased over the past years.

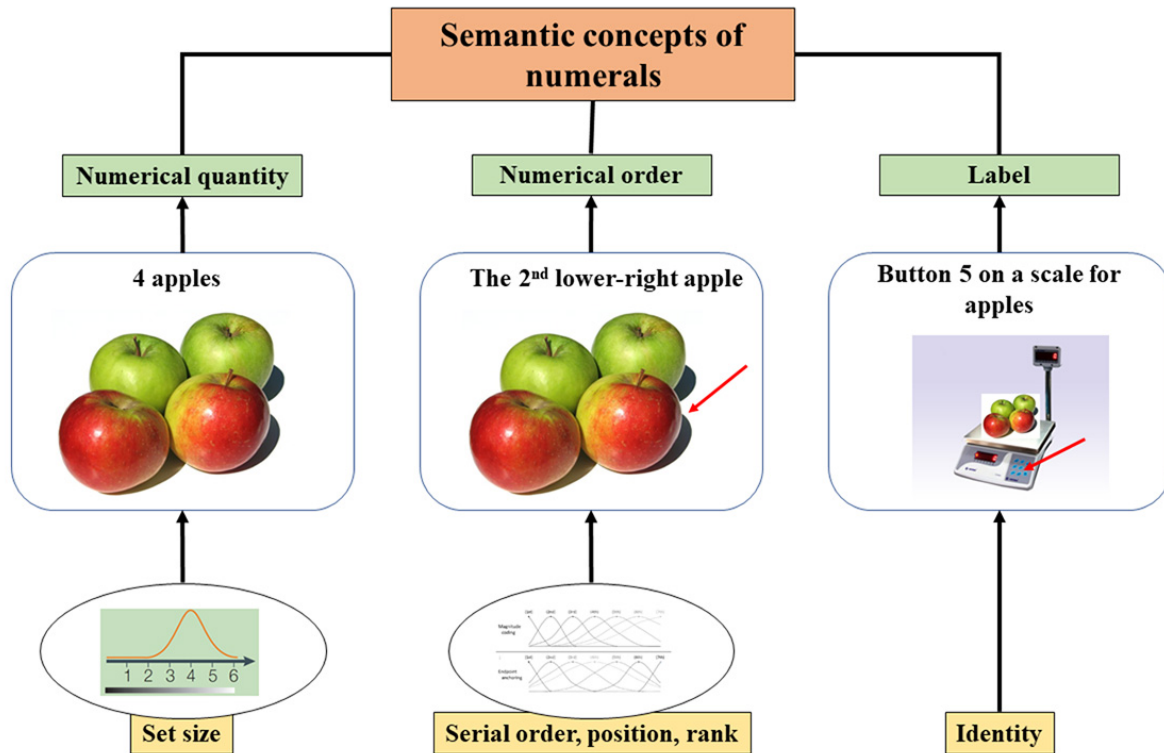


Figure 1. Schematic illustration about the different concepts that numbers refer to (adapted from ref. [5]).

Findings from this work have shown that numerical order is linked to a good understanding about the internal structure and meaning of number sequences. For instance, children need to understand the stable-order-principle of number words: that is, that a particular number word such as */nine/* always comes after the number word */eight/* and before the number word */ten/* in the counting sequence<sup>[7,8]</sup>. This knowledge builds the foundation for the development of the so-called successor and predecessor function. These functions denote a general understanding that every number in the counting list has a predecessor ( $n-1$ ) and a successor ( $n+1$ ), and that the next counted object corresponds to the next number word in the counting list<sup>[9,10]</sup>. Over developmental time, knowledge about the internal structure of number sequences and how numbers relate to each other becomes highly familiar and automatized. This automatization is crucial for the development of arithmetic abilities<sup>[4,11,12]</sup>.

### How is numerical order associated with arithmetic abilities?

How do we investigate numerical order processing in children? One task that is frequently used is the numerical order verification task. Children (or adults) are asked to decide as quickly and as accurately as possible whether numerals on a computer screen are displayed in an ascending/descending order (e.g., 2-3-4) or not (e.g., 3-5-4). Reaction times and error rates are measured as children perform this task. The behavioral measures (reaction time and accuracy) show a distinctive signature of numerical order processing: the so-called *reverse distance effect*<sup>[11]</sup>. Number sequences with a numerical distance of one (e.g., 2-3-4) are processed much faster and with fewer errors than sequences with a numerical distance of two (e.g., 2-4-6). Research indicates that the reverse distance effect is a direct measure of how efficiently numerical sequences are processed<sup>[11-13]</sup>. Number sequences that are close together in space (e.g., 1-2-3) are processed more efficiently than numbers that are farther apart in space (e.g., 3-6-9). Importantly, there is substantial individual variation in the size of the reverse distance effect. Some people show a large effect, while others show a small effect. It appears that some individuals are more efficient in processing numerical order than others. But what does this mean and how is this important for arithmetic?

Recent behavioral findings indicate that individual differences in the efficiency to retrieve numerical order is related to arithmetic abilities. In other words, children who demonstrate more efficient processing of numerical order also demonstrate better performance in arithmetic: They solve arithmetic problems more accurately and faster within a given time interval<sup>[12,14-16]</sup>. Developmental studies further demonstrate that this association increases over developmental time<sup>[15,17]</sup>. For instance, a large study investigated the relationship of different numerical abilities, including numerical order, and arithmetic

abilities across 1,391 children in grade 1 to 6<sup>[15]</sup>. The results of this study showed that the relationship between numerical order increased over the school years and became the best behavioral predictor of arithmetic abilities in 4<sup>th</sup> grade. Numerical order was even a better predictor than knowledge about *numerical magnitude*. In other words, children who are proficient in judging the sequential order of numerals were also really good in arithmetic skills.

New findings also indicate that numerical order has a central role in mediating the relationship between *numerical magnitude* and arithmetic<sup>[17,18]</sup>. In other words, the initial importance of the relationship between *numerical magnitude* and arithmetic skills seems to be transmitted via numerical ordinal knowledge. While the reasons for the relationship between numerical order and arithmetic are still debated, brain imaging studies show some preliminary evidence for why this is the case.

### How does the brain process numerical order?

The question of how the child's brain processes numerical order and how the brain responses of numerical order differ from numerical magnitude is currently at the forefront of neuroscientific investigations. Two *functional magnetic resonance imaging (fMRI)* studies with children recently demonstrated that regions of the left inferior parietal cortex (see Figure 2) appear to increase their activity in response to numerical order processing with age<sup>[19,20]</sup>. In other words, this brain region becomes more specialized for the processing of numerical order as children get older. This particular finding of greater activation in the left hemisphere of the parietal cortex is in line with other functional imaging studies that demonstrated that the left intraparietal sulcus processes the mental representation of number symbols<sup>[21,22]</sup>. Specifically, this brain region shows particularly strong activation when participants deal with numerals or number words.

The studies also found that the brain activity differs between numerical order and *numerical magnitude* processing. This differential effect means that different brain mechanisms seem to be involved when children engage in numerical order or *numerical magnitude* processing. The results converge with adult studies that also demonstrated different brain activation patterns between numerical order and numerical order processing<sup>[23,24]</sup>. These findings are crucial since they indicate that numerical order is not just an epiphenomenon of numerical magnitude processing, but rather constitutes an independent dimension of numerical knowledge.

Another finding from this work is that the association between numerical order and arithmetic might be related to the engagement of the semantic control network<sup>[20]</sup>. The semantic control network includes a number of different brain regions in the frontal and the parietal cortex (see also Figure 2): the inferior frontal gyrus (IFG), the posterior middle temporal gyrus (pMTG), the intraparietal sulcus, the pre-supplementary motor areas and the anterior cingulate cortex. These brain regions are thought to be *domain-general*—they are not specific to a particular domain (e.g., mathematics), but are relevant for a broad range of different control functions. These regions interact with *domain-specific* presentations—that is, brain processes that are relevant to a specific domain of interest (e.g., number representation) to engage in control processing mechanisms to perform a specific task<sup>[25]</sup>. These mechanisms might explain the transmitting role of numerical order discussed in the section above.

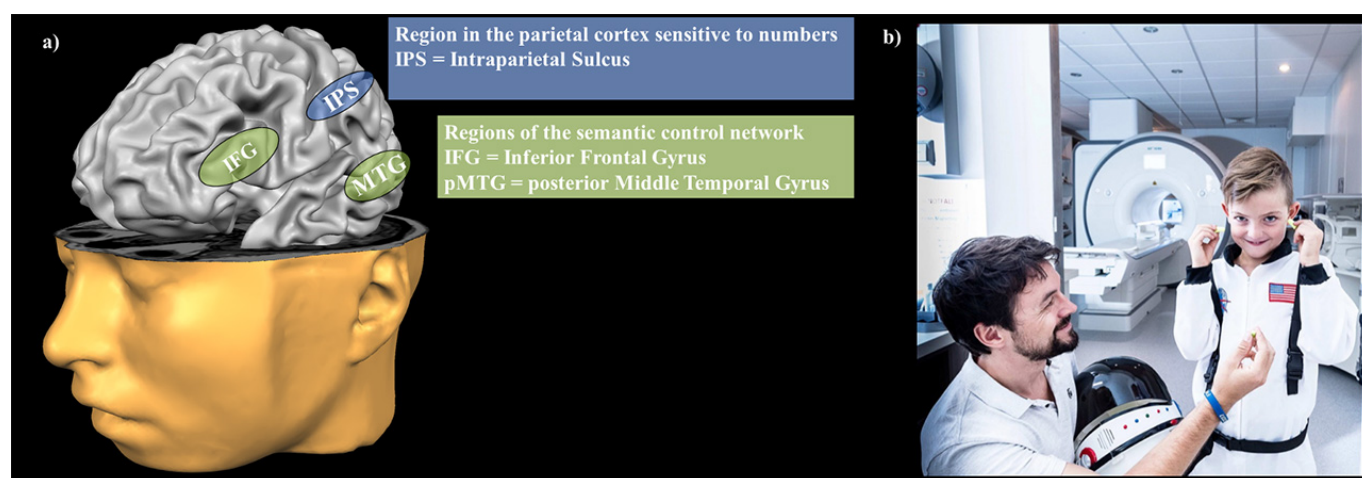


Figure 2. (A) Brain regions associated with numerical order processing in children. (B) Child taking part in one of our fMRI studies.

In addition to these key findings, there is emerging evidence that children with developmental dyscalculia demonstrate difficulties in understanding numerical order and that their brain responses differ from those of their typically developing peers<sup>[26,27]</sup>. In one study, the brain response during a numerical order task was measured in a group of children who were diagnosed with developmental dyscalculia and a group of children who had no difficulties in math<sup>[26]</sup>. The results of this study showed significant differences in brain activation in regions of the parietal cortex. It thus provided further evidence that regions of the parietal cortex are involved in numerical order processing and that children with learning difficulties show systematic differences in brain activation in this region.

Together, the findings imply that numerical order is a crucial dimension for the development of arithmetic abilities. Although the precise processing mechanisms are still not fully understood, we have started to gain a better picture of the mental processes that are involved. This understanding will help us to further identify core functions associated with mathematical abilities and to enhance the structure of mathematical learning materials.

### What are the implications for math education?

Fostering the development of mathematical abilities encompasses the integration of multiple aspects associated with symbolic number knowledge<sup>[28]</sup>. These range from numerical magnitude processing, spatial number knowledge (see also the article [“The role of spatial skills in early math learning and how to foster space-number interactions”](#) on this website), to knowledge about numerical order. Unfortunately, many children lack knowledge in these central dimensions due to limited exposure to numbers and their relationships. Preschool and home environments often do not provide sufficient support for young children’s mathematics development. A crucial theme to empower children’s numerical learning is, therefore, to provide ample access to structured materials that help children develop a good understanding of number representation. Although caution should be exercised when attempting to translate findings from neuroscience to practices in education, I will aim to point out several important aspects that appear to be of broader relevance for children who struggle with numbers.

First, findings from neurocognitive research can inform and refine the definition of children with special educational needs—such as children with developmental dyscalculia. In many countries, financial and specific support for an individualized intervention is restricted to an official diagnosis<sup>[29]</sup>. Providing the scientific basis to better define learning difficulties has, therefore, important consequences for support structures that aim to help children with mathematical difficulties. For instance, understanding that the knowledge of numbers involves distinct representations—such as numerical magnitude and numerical order—with a biological basis helps to further implement specific definitions to better help those who are in need.

Second, it is of paramount importance to implement tools that assess core dimensions of numerical abilities to identify children at risk. The current evidence indicates that good knowledge of numbers in kindergarten is a powerful predictor for arithmetic achievement in schools<sup>[30]</sup>. The assessment of early numerical skills might turn out to be a central component for effective math interventions. This argument is substantiated by several studies from the domain of reading that show that the optimal window for *dyslexia* intervention is in kindergarten before school onset<sup>[31]</sup>. Although strong evidence is still missing in the domain of math, the same might hold true for children with mathematical difficulties<sup>[32]</sup>. A good example for an efficient screening tool (derived from neurocognitive findings) to assess early numerical abilities is the numeracy screener<sup>[33]</sup>. This paper-and-pencil test measures symbolic and nonnumerical core abilities (<http://www.numeracyscreener.org/>) in young children.

Third, the development of structured and specific intervention tools is crucial when working with children who have learning difficulties in math. The results of a large study that summarized the outcome of 58 studies on the effectiveness of math interventions showed: (1) that individual interventions are more effective than group interventions, (2) that fostering basic numerical abilities is more effective than teaching math problem solving skills, and (3) that short intervention periods lead to relative greater gains in mathematical abilities compared to long intervention periods<sup>[34]</sup>. The latter might be due to the structured content that is used in shorter learning sessions. This further indicates the necessity to construct specific learning materials that build upon central dimensions of numerical knowledge as identified by neuroscience.

Finally, as indicated above, the successful development of arithmetic abilities is linked to the development of several numerical dimensions—such as numerical order. Methods that highlight the structure of these numerical dimensions constitute powerful tools to foster numeracy in children. Children, especially those coming from families with low socioeconomic status, often lack the opportunities to interact with materials that have a structured numerical content<sup>[32]</sup>. A useful method to empower these children are board games, in which the structure of numerals is built into the games—such

as the well-known game *Chutes and Ladders* (also sometimes known as *Snakes and Ladders*). Research has shown that the linear structure of numbers in these board games enhances the quality of numerical representations in children<sup>[35]</sup>. Overall, in order to harness the full potential of our next generations, schools and educational systems need to be better equipped with empirically tested tools that assess and foster the development of mathematical abilities.

### Key points for education

- Implementation of structured materials that highlight different symbolic numerical dimensions in game-like formats.
- Provide specific learning materials for children who do not have access to these materials.
- Build a solid foundation of core abilities before more complex concepts of mathematics are introduced.
- Implementation of assessment tools that measure individual differences in central core abilities to identify and diagnose children with mathematical difficulties.
- Provide resources that allow systematic and individual help for children with special education needs within an "optimal window for intervention."

### Glossary

**Domain-general** – Brain processes that are not specific to a particular domain (e.g., mathematics), but are relevant for a broad range of different cognitive functions.

**Domain-specific** – Brain process that are specifically relevant to a particular domain of interest. In mathematics they mostly comprise different aspects of number processing (e.g., representation of numerical magnitudes, knowledge of the ordinal relationships of numbers).

**Dyslexia** – Specific learning disability in the domain of reading. Children with dyslexia show severe difficulties to map graphemes (i.e., visual representation of letters) to phonemes (i.e., speech sounds of letters) and to acquire reading skills.

**Functional magnetic resonance imaging (fMRI)** – Neuroscientific method that is used to infer brain activation from subtle changes in the blood oxygenation level in a particular brain region. Brain regions that are active consume more oxygen to sustain their activity. A strong magnetic field is used to detect these subtle changes in oxygen due to brain activation.

**Neurocognition** – Cognition is defined as "mental action or process of acquiring knowledge through thought, experience, and the senses"<sup>[36]</sup>. Neurocognition relates cognitive functions to the function of particular areas, neural pathways, or cortical networks in the human brain.

**Number comparison task** – An experimental task that is used to investigate numerical magnitude processing. Participants are instructed to decide as fast and as accurate as possible which of two presented numerals is larger or smaller.

**Numerical distance effect** – A behavioral response pattern that is characterized by faster reaction times and less errors as the numerical distance between the two numbers being compared increases.

**Numerical magnitude** – Knowledge about the quantity (i.e., the number of items in a set) that is associated with a numeral.

[1] The reverse distance effect can be differentiated from the **numerical distance effect** that is typically observed during **number comparison tasks** and which is associated with **numerical magnitude** processing.

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