Mathematics anxiety and mathematics performance

Mathematics is often perceived as a difficult subject by students, parents, and teachers alike. For many, it is a source of anxiety that interferes with the mathematics itself. Yet difficulties with mathematics are most often attributed to cognitive reasons (lack of ability, preparedness, practice, and knowledge), and emotional factors are ignored.

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Theme/s:

Effective teaching / Learning mathematics / Learning difficulties / Emotions and learning / Learning how to learn

This report arises from Science of Learning Fellowships funded by the International Brain Research Organization (IBRO) in partnership with the International Bureau of Education (IBE) of the United Nations Educational, Scientific and Cultural Organization (UNESCO). The IBRO/IBE-UNESCO Science of Learning Fellowship aims to support and translate key neuroscience research on learning and the brain to educators, policy makers, and governments.

Executive summary

- Not all mathematics difficulties stem from cognitive problems.
- Many pupils and adults have a debilitating emotional reaction to any situation involving mathematics: mathematics anxiety.
- Mathematics anxiety is distinct from other forms of anxiety.
- Mathematics anxiety negatively correlates with mathematics achievement.
- Mathematics anxiety likely disrupts working memory processes crucial for mathematics.
- Mathematics anxiety may be a component of a vicious circle, anxiety leading to worse performance and performance decrement justifying anxiety.

Mathematics anxiety: Overview

Mathematics is often perceived as a difficult subject by students, parents, and teachers alike. Difficulties are most often attributed to cognitive reasons (lack of ability, preparedness, practice, and knowledge). In contrast, emotional factors are rarely considered or are easily written off as potentially persistent causes of mathematical difficulties. However, it is increasingly recognized in psychology and education that many students have serious emotional problems with mathematics that inhibit their performance and progress in this domain. In particular, many students have a debilitating emotional reaction to mathematics termed **'mathematics anxiety'** (**MA**).

Anxiety can be defined as the "feeling of uncertainty and helplessness in the face of danger"[1]. MA has been **defined** as "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in…ordinary life and academic situations"[2-3]. MA ranges from the feeling of mild tension to experiencing strong fear of mathematics. MA can manifest itself in different ways, for example, as feelings of apprehension, dislike, tension, worry, frustration, and fear[4-6].

MA as a specific anxiety is likely to be an acquired, **socially constructed** anxiety that is not restricted to test situations or classroom settings. Hence, while MA is typically detected in school environments, it can also appear in any situations involving mathematics (e.g., when handling numbers in shops or needing to solve calculations in everyday life). That is, MA can generalize to various situations. An important consequence of MA may be that otherwise perfectly intelligent and mathematically capable persons develop a severe *avoidance* of situations involving any kind of mathematics and *do not choose careers* involving the application of mathematics[7].

MA disproportionally affects **girls** and, therefore, may contribute to the worldwide observable gender gap in science, technology, engineering, and mathematics (STEM) subjects. In addition, MA may also contribute to girls' *stereotypical career choices* and career aspirations[8-9], contrary to the fact that they often show equal or better mathematical performance than boys[10].

Measurement of MA: Questionnaires

Academic anxieties such as MA are **not** considered clinical anxiety disorders. Similarly, academic anxieties are currently not recognized in the *Diagnostic Statistical Manual of Mental Disorders* (DSM–5; American Psychiatric Association, 2013*),* nor in the *International Classification of Diseases (*ICD–10; World Health Organisation, 1992). In fact, in clinical practice, questionnaires alone cannot be used to diagnose specific phobias (American Psychiatric Association, 2013; World Health Organisation, 1992). Nevertheless, questionnaires are used extensively in educational and psychological research for identifying MA and test anxiety in children and adults.

The Maths Anxiety Rating Scale (MARS) is a frequently used adult MA questionnaire[11]. The so-called Abbreviated Maths Anxiety Scale, abbreviated **AMAS**, a 9-item questionnaire, has been developed based on MARS[12]. A thorough overview[4] suggested that AMAS could be a major tool for adult MA research as it is brief and delivers results consistent with the much

longer MARS. The AMAS has been translated into various languages. There have been reliable and valid Persian, Italian, and Polish translations of the AMAS[13-15].

Importantly, the AMAS contains questions that are not suitable for primary school children. For example, some questions refer to looking up information in tables in the back of mathematics books. However, such tables are not typical in primary school mathematics books. Yet, some other alternative questionnaires[16] may include too specific questions (e.g., about specific addition results such as 13 + 6). However, such questions are again not very suitable for reliably determining MA. First, younger children may find them more difficult than older children. Second, mathematically underperforming children may find such concrete questions more anxiety-inducing than more advanced students. However, *some* of the more advanced students may actually feel substantial MA when encountering more difficult problems, whereas other advanced students may not feel much MA. Taking into account these considerations, it is imperative to develop a reliable (consistently performing from time to time) and valid (measuring a well-defined construct) instrument for measuring MA in young children[16].

Carey et al.[16] have developed the modified AMAS (**mAMAS**) questionnaire **suitable for primary and secondary school children**. The study used data from 1,746 British children and found that the mAMAS is a good measure of MA in children. The mAMAS has two subscales, Learning MA and Evaluation MA[12]. Both scales can be considered to be part of the same MA construct. The mAMAS is a freely available open access resource published in the Supplementary Material of Ref. [16].

Notably, while the AMAS and the mAMAS differentiate between the above-noted Learning and Evaluation subscales, other MA scales may define different subcomponents. For example, some researchers separate MA elicited by testing situations from different types of MA (e.g., anxiety for doing arithmetic, using mathematics in everyday life, or manipulating numbers)[17]. Others[18] supposed that MA may have two dimensions: (1) a cognitive component (usually labelled as "worry") that concerns worries about performance, and (2) an affective component ("emotionality"), which refers to tension, nervousness, and related physiological reactions felt in evaluative settings[19].

Measurement of MA: Neuroscience and implicit measures

A general problem with all kinds of questionnaires is that they are based on self-report. Hence, questionnaire results may be affected by metacognitive processes, self-knowledge, and cultural biases. It is an important goal of educational neuroscience to develop **objective measures** of MA. Such measures could be based on physiological parameters such as cortisol levels, functional magnetic resonance imaging (fMRI) data, electro-physiological or electro-myographic indices, and eye-tracking parameters. Alternatively, implicit psychological measures (that do not require overt, conscious deliberation on behalf of pupils) could also be used in principle, such as implicit priming measures[20]. At the moment, the practical applicability of physiological and priming measures is not yet clear. More research is needed to determine their validity and reliability.

Prevalence of MA

Prevalence estimates of MA depend on the definition of high MA levels based on questionnaire measurements. Inevitably, all such definitions are subjective and are based on researcher judgement. Importantly, MA researchers use discrepant definitions of what constitutes high MA. Some have defined high MA as achieving MA scores higher than +1 standard deviation (SD) above the mean[21]. This definition of high MA assumes that MA scores are normally distributed. Hence, a cutoff of +1 *SD* above the mean would imply that about 17% of children would have high MA. However, the distribution of MA scores is typically not reported. In fact, recent large population investigations of MA suggest that MA scores are not normally distributed[16]. Therefore, the +1 *SD* definition of high MA is questionable.

A recent large investigation[16] determined the distribution of MA scores in the United Kingdom in more than 1,700 children aged 8 to 9 years (primary school) and 12 to 13 years (secondary school). The study used the above-discussed mAMAS questionnaire. The distribution was non-normal (see Figure 1 in Ref. [16]). Hence, arbitrarily, researchers considered children to have high MA if they had a score higher than the 90[th] percentile of the score distribution. In other words, this means that the **top 10% most math-anxious** children were considered to have high MA. This definition meant that in order to be classified as highly math-anxious, children had to achieve a score of 30 or higher out of a maximum score of 45 on the mAMAS. Notably, this 30 cutoff score for high MA definition entirely depends on the empirically observed data distribution. Hence, this cutoff score can be considered empirically validated.

MA is distinct from other forms of anxiety

MA is positively correlated with so-called **test anxiety**, elicited by testing situations ([22]; *r* = 0.52). However, a major review found that 63% variance in MA cannot be explained by test anxiety[22] and MA is typically considered a distinct construct from test anxiety[4]. Moreover, as MA is one of the most prevalent academic anxieties in childhood, many test anxiety questionnaires may actually measure MA primarily. This is because when asked about their own academic anxieties, many children may primarily think about their negative experiences and anxiety in mathematics classes. Hence, a correlation between MA and test anxiety does not allow us to conclude which anxiety form is "primary."

Another anxiety form that has been shown to relate to MA is **general anxiety.** General anxiety refers to an individual's tendency to feel anxious about everyday situations. General anxiety often has factors of physiological anxiety, worry, and social anxiety (see, for example, the Revised Children's Manifest Anxiety Scale[23]). A large study of 800 Italian children found substantial shared variance between general anxiety and MA[24]. Nevertheless, this correlation was less strong than the correlation between test anxiety and MA.

One large recent investigation in the United Kingdom aimed to **differentiate** between various **anxiety forms and their role** in relation to academic performance[25]. The study found that younger (8- to 9-year-old) children's MA, test anxiety, and general anxiety levels were very similar. That is, MA tended to appear strongly linked to other forms of anxiety. Put otherwise, children who were generally anxious were also likely to experience high levels of test anxiety and MA. In contrast, older (11- to 13-year-old) children showed more **specific forms** of anxiety with more **specific impact** on academic performance. That is, some children may have been generally anxious, but others now showed only *specific* MA or anxiety to testing situations. Moreover, the generally more anxious children had better math performance than the children with specific MA. Hence, there seems to be a developmental change in the pattern of various anxieties during the school system: It seems that specific academic anxieties become more and more pronounced during schooling and that specific academic anxieties become linked to poorer academic performance more than general, nonacademic anxieties.

MA has a negative correlation with mathematics performance

Meta-analyses of many studies have found that MA has a moderate correlation of about r = -0.27 to r=-0.34 with mathematics performance[22,5]. Note that the negative sign indicates that high MA scores are associated with lower mathematics performance. These estimates are in perfect agreement with a recent large population study that measured r = -0.3 correlation (95% confidence interval = -0.34 to -0.25) between MA and standardized math achievement scores in more than 1,700 primary and secondary school pupils in the United Kingdom[26]. These correlations are relatively low, but they do indicate a consistent relationship.

There is evidence that MA can already negatively affect performance in 5- to 7-year-olds[27]. However, a recent study with more than 800 Italian children observed lower effect sizes of r = -0.13 and r = -0.07 correlations between MA and math performance in primary school girls and boys respectively[24]. In contrast, correlations were comparable to the above-noted effect sizes (r = -0.34 and r = -0.28) in secondary school girls and boys. These observations appear to be in line with the above-discussed proposal that MA and academic anxieties become more linked to academic performance during schooling. That is, at least in some countries, MA and negative attitudes toward mathematics may increase during schooling, may reach adult levels at the secondary school level, and then persist into higher education and adulthood [see Ref. [25] for extended discussion].

While girls show overall higher levels of MA than boys (see author's brief, "The origins of math anxiety and interventions"), the amount of correlation is the same in both genders (that is, the slopes of bivariate MA versus math performance fitted lines are the same in both genders).

Theories of the negative impact of math anxiety on math performance

According to the so-called **deficit theory**, MA is the *consequence* of poor math performance[28]. That is, children would expect themselves to perform poorly in mathematics tests and would therefore predict/expect poor performance on these tests. This prediction would then elicit anxiety and act as a self-fulfilling prophecy. Importantly, according to this view, *only* children with poor math performance would develop MA.

According to the so-called **interference theory of test anxiety**[29], people in a situation would experience both *task-directed* drives and *anxiety* drives. Task-directed drives would help in fulfilling task requirements and completing required jobs. In contrast, anxiety drives may result in two different behaviours. First, if they are not overwhelming, they can trigger task-

relevant efforts that can result in reduced anxiety levels and trigger task-relevant drives/behaviour. Second, if anxiety drives become overwhelming, they can trigger self-directed, task-irrelevant behaviour that interferes with task performance. Pupils can experience heightened heartbeat and may anticipate punishment, loss of status, or loss of self-esteem. They may also experience a strong desire to escape the negative situation. If the anxiety drives become very strong, pupils may develop a habit of reenacting task-irrelevant behaviours, which may result in long-term performance decrement.

The **debilitating MA** model builds on the above **interference** model and assumes that relative low performance in mathematics is a consequence of high MA rather than vice versa[30]. Basically, pupils would experience anxiety which would trigger task-irrelevant behaviours consuming working memory resources, which in turn would decrease mathematics performance. Put simply, children would think about many task-irrelevant potential negative outcomes (e.g., failing the test, being unable to solve the test, others performing much better than them, being punished at home for bad marks) and these irrelevant thoughts would then heavily compromise working memory performance. As discussed in **Brief 1**, working memory is a vital mental space where mathematics operations can be carried out. Hence, it is feasible to assume that anxiety consuming working memory resources may be able to cause performance decrements in mathematics[31-32].

Evidence suggests that MA negatively affects performance primarily on **more difficult than easier mathematics problems** [32]. For example, Ashcraft and Krause gave mathematics problems to adults with varying levels of MA. People with the highest levels of MA had lower performance on medium-hard and hard problems than people with low levels of MA. The findings are consistent with the working memory interference explanation of MA because higher levels of working memory performance would be needed to solve more difficult than easier problems.

Importantly, the deficit theory and the debilitating anxiety theory of MA are **not** mutually exclusive[28]. They may both be valid factors: 1) being valid in **different groups** of pupils, or 2) being valid at the same time as **mutually contributing** factors to amplifying MA.

First, it was suggested that some pupils may have generally high anxiety levels to start with and may consequently also be predisposed to develop high MA irrespective of their math performance[25]. That is, in this group of pupils, the debilitating anxiety model may be more valid. In contrast, other pupils seemed to have only specific academic anxieties, and they also showed the lowest relative levels of mathematics performance. That is, in this group, the deficit theory may be more valid.

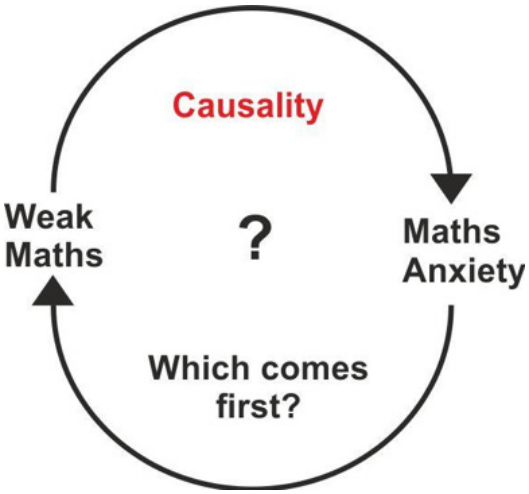

Figure 1. Reciprocal theory. A vicious circle between debilitating MA and deficit-induced anxieties. A major question is whether weak mathematics performance or anxiety may come first in starting the vicious circle. Different factors may be primary in different groups of children.

Second, the so-called **reciprocal theory**[28] considers the option of both debilitating anxiety and deficit-induced anxieties being in play at the same time. Figure 1 demonstrates a potentially **vicious circle** of debilitating anxiety and deficit-induced anxiety amplifying each other during development. On the one hand, lower ability children may experience anxiety due to their low perceived performance on math tasks. This perception may then induce debilitating anxiety in subsequent tests. This

debilitating anxiety would then result in further performance decrements. On the other hand, normally achieving children would first experience debilitating anxiety. This anxiety could result in performance decrement in mathematics test that would induce (perceived) deficit-related anxieties. These could then further justify pupils' debilitating anxieties as well.

In the medium term, such anxieties would keep children away from home-based, extracurricular, and optional math education opportunities that would cause further relative (to others) performance decrements. Hence, a vicious circle between debilitating and deficit-induced anxieties may be in play in a portion of children.

More MA studies are needed worldwide

Most studies of MA have been carried out in the USA and United Kingdom[22,4]. Worldwide coverage of research is lacking, especially in developing countries. However, the existing studies suggest that MA is as important, if not a more important factor, in mathematics learning and gender discrepancies in learning in developing countries[33-34].

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