Mathematical learning difficulties

Numerous behavioral and neuroimaging studies suggest that mathematical learning difficulties, are related to weak performance on short-term memory and working memory tasks.

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Executive Summary

- Selective cognitive Mathematics Learning Difficulty (MLD) is diagnosed when children are unable to progress in mathematics but their performance is adequate in other domains.
- There is no reliable gender difference in MLD.
- MLD likely has various subtypes.
- At least some types of MLD are associated with short-term and working memory weaknesses.
- On type of MLD is characterized by weak visuo-spatial working memory and is not associated with reading problems. Another type of MLD is associated with reading problems and is associated with weak verbal working memory.
- Memory weaknesses are likely associated with abnormal structure and function of the Intraparietal Sulci (IPS) of the human brain.

Numerous behavioral and neuroimaging studies suggest that mathematical learning difficulties (MLD), are related to weak performance on short-term memory (STM) and working memory (WM) tasks. This document reviews evidence for this conclusion, details theoretical background and points to educational implications.

Terminology and definition of Mathematical Learning Difficulties

An important note regards terminology. In the cognitive neuroscience and psychological literature mathematical learning difficulties are often called Mathematical Learning Disabilities (also abbreviated MLD) or developmental dyscalculia (DD). Here I deliberately avoid using 'Disability' as label as in my opinion there is not much evidence to suggest that learning weaknesses are associated with irreversible conditions whereas the label 'Disability' often conveys this implication. The irreversible, 'disability view' of mathematical weaknesses is linked to ideas that assume that biologically based cognitive skills may be irreversibly impaired in some children. However, I do not think that much evidence supports the biologically based irreversibility of mathematical weaknesses (see e.g. Brief 2 about problems with the biologically based 'number sense' theory of mathematical development). Hence, in this document I will use the more neutral 'Mathematical Learning Difficulty' term. Coincident with a large portion of the literature this also abbreviates to MLD.

To date researchers do not have an agreed upon definition of MLD. Some researchers consider 'mathematical learning disabilities' and DD to be distinct mathematical learning problems whereas others consider them as different labels for the same underlying mathematical learning problem. The terminological confusion appears because currently there is no generally accepted nomenclature of developmental mathematical problems.

Here, the MLD term will be used in a sense, as we defined DD previously: MLD will stand for 'persistently weak mathematical performance of developmental origin, related to the weakness of some kind(s) of cognitive function(s) and/or representation(s); appearing when *concurrent* motivation to study mathematics and access to appropriate mathematics education is normal' [1; p1].

In practice, in a theory free manner, children are typically considered to have MLD if they show mathematical performance weaker than a certain criterion level under the mean performance of a standardized mathematics achievement test. The criterion level is often one standard deviation or one and half standard deviation under the mean level. Importantly, there is no a priori reason to use a certain criterion level so they may differ widely from study to study.

Further, some studies have tested whether mathematical performance is specifically weak in children by using a control criterion test. For example, researchers may also test reading performance in children and they only categorize children to have MLD if their mathematics performance is weak, but their reading performance is in the normal range of age appropriate achievement [2]. However, many researchers do not explicitly test for the specificity of mathematics problems. Importantly, researchers do not agree in what measures should constitute adequate control variables. Some used reading performance, other used various intelligence (IQ) measures.

It is important to realize that some variability of findings in the MLD literature appears because different researchers use different parameters for the above variables: they may use different control variables and cutoff scores in the diagnosis of MLD. Hence, they select slightly or very different participant groups for study and this results in diverse findings. Hence, it is important to carefully evaluate and compare the exact operational definitions of MLD used in each particular study.

The simplest disagreement concerns prevalence estimates of MLD. For example, a thorough review of many MLD prevalence studies found that prevalence estimates ranged between 1.3-10.3% and their mean was about 5-6% [2]. In view of the above, the large variation in prevalence estimates likely depends on the exact diagnostic measures and thresholds used in studies.

There is no gender difference in MLD

Because of its practical relevance it is important to comment on potential gender differences in MLD. Some investigators suggested that MLD may be more prevalent in girls. This suggestion gave rise to theories linking MLD to X chromosome-based dysfunctions [3-6] (women have two X chromosomes while males have an X and an Y chromosome). However, these studies were based on specially selected clinical populations and cannot be considered representative of the whole population. For example, many studies were based on observations from individuals with fragile X syndrome and Turner syndrome that are rare genetic disorders [3-6].

In contrast to the above-mentioned small studies based on rare populations, large population based empirical studies using standardized mathematics and control (reading) measurements could not find evidence in support of a gender discrepancy in MLD [3,7]. These studies investigated the use of multiple different cutoff points for the diagnosis of MLD and concluded that no gender difference exist independent of the cutoff points used. These results are also in line with a large genetics study of 10-year-old children that concluded that there were no genetically based gender differences in mathematical ability in 10-year-old children [8].

Importantly, gender differences may appear if MLD is defined as a discrepancy between standardized reading and mathematics performance. However, in this case gender differences typically appear not because girls are worse in mathematics than boys but because girls tend to be better readers than boys whereas their mathematics performance is similar [3,7,9-10]. Hence, a reading minus math discrepancy score will show larger discrepancy in girls than in boys penalizing girls with relatively good reading skills. Therefore, reading vs. mathematics discrepancy measures are not adequate for diagnosing MLD.

Short-term and working memory and its neural substrates

The most popular working memory model in MLD research is the classical model of Baddeley [11]. This model assumes that memory function involves modality specific, verbal and visual short-term memory (STM) stores and a domain-general central executive (CE) processing unit. STM is the ability to maintain information in unchanged format for a short while. Working memory would refer to the ability to maintain information while simultaneously carrying out some operation on the maintained information (see Brief 2 for examples). That is, STM tasks only require the mere maintenance of information. In contrast, in WM tasks a secondary, so-called *processing* task is also carried out besides maintenance. Baddeley's model assumes that the processing task relies on STM function and on the involvement of the so-called 'central executive' function. A version of the model supposed that CE function relies on a limited capacity attentional control system [12].

Central Executive function can be defined in more detail. Many researches assume that various, so-called *Executive* functions contribute to CE performance. First, it is important to emphasize that there is still no clear definition of executive functions. Hence, 'executive functions' is a large umbrella term and its exact definition may vary from study to study. Nevertheless, there seems to be agreement that the followings are important executive functions that also play a role in memory performance: Inhibition; attentional focus shifting and information updating and monitoring [13]. *Inhibition* typically refers to the suppression of unwanted inference from processed items and it has been emphasized by several investigators [14]. *Updating* refers to the function when items initially in the focus of attention must be overwritten after becoming irrelevant and a new item should enter the focus of attention. *Shifting* is typically assessed in non-memory tasks that require volitional control [15].

Working memory and MLD

Various aspects of working memory function can plausibly be thought to be important for mathematical function and development. Solving even a trivial equation with very small numbers, for example, $((3+4)-(2-1))/(2\times2)$, requires a substantial amount of planning. In fact, even adults are unlikely to be able to solve the above simple equation 'in an instant', they have to carefully decompose it, make a plan of how they want to proceed with solving the entire equation, direct their attention to individual parts, solve parts one by one, keep partial results in mind as long as they solve parts of the equation and finally integrate all parts in a final mathematical operation and keep the result in mind as long as they communicate it in some form. Indeed, several studies have reported working memory problems in MLD. Here we have space only to review a few representative studies.

One series of experiments investigated verbal STM and working memory problems in MLD. A very early study [16] concluded that children with MLD were only impaired in remembering numbers but not in general working memory. However, later studies [17] found that while MLD children may indeed have specific weaknesses in so-called forward and backward digit span STM tasks (in these tasks children have to remember series of numbers and have to recite them in the original or reversed order) they also had a general working memory deficit reflected in poor performance in many other verbal working memory tasks.

A follow-up study [18] tested 22 MLD children and 27 control children. The paper replicated the general verbal working memory deficit in MLD children and also reported preserved verbal STM in MLD. That is, only working memory tasks requiring executive function contributions were impaired. In addition, children committed many so-called intrusion errors in memory tasks. This means that irrelevant information influenced their task performance. Based on the intrusion errors investigators concluded that the primary deficit in MLD was the impairment of the inhibitory component of central executive function contributing to working memory performance. Various other papers also concluded about the importance of inhibitory function in MLD [19-20].

While many studies used verbal measures of memory, relatively few studies measured visual memory in MLD children. One such study [21] tested both visual and spatial STM in DD, dyslexic, MLD+dyslexic and normal populations and found only visual STM impairment in MLD and only verbal STM impairment in dyslexics. The results of this study suggested that when reading and verbal function was preserved a crucial problem in MLD may concern visuo-spatial memory processes.

Overall, the above results suggest that various aspects of working memory function may be impaired in MLD. A difficulty in integrating results is that many studies did not use an extended battery of working memory tests. In order to gain a comprehensive view of possible memory function deficit in MLD Szucs et al. [22]. have tested both verbal and visual STM and verbal and visual working memory in a very carefully tested and selected population. All together, researches used 18 standardized tests, 9 experimental tasks and various other measures with each child. MLD children were tightly matched to control children in that only the mathematical performance of children but not their reading performance or intelligence differed. Findings showed that MLD children had weaker visual STM and working memory than control participants (The paper used so-called visual matrix span tasks where children are shown a grid with dots occupying grid positions. Children have to recall the position of dots. In another version the dots appear one by one and children also have to recall the order in which dots appeared [24].) In addition, MLD children also had weaker inhibition performance than control participants in various different experimental tasks measuring inhibition [25-26]. It is important to note that this paper could not find any evidence to support claims that an impairment of a putatively biologically based number sense (see Brief 2 and [27]) plays any role in MLD.

MLD and working memory: Subtypes of MLD

As described above, a very large number of studies reported working memory weaknesses in MLD. Overall, a confusing array of results was obtained. Specifically, different studies reporting weaknesses in different memory systems. For example, some studies found verbal but not visual working memory problems and vice versa. Yet, other studies found weaknesses in certain executive functions supporting working memory performance while different studies could not confirm the results. First, it is possible that different participant selection criteria affected results (see above). Second, it is possible to find some variables that help organize the results.

A review of 32 often cited MLD studies investigating short-term and working memory performance determined that an important organizing factor to consider was whether the reading performance of children with MLD and that of control children were similar [28]. Importantly, in many studies children with MLD were not only weak in mathematics but they were also poor readers. That is, they were likely to have additional dyslexia and/or other reading comprehension deficits on top of their mathematical difficulties. In contrast, other studies examined children with MLD who were reading at an age appropriate level. The literature review also considered the effect of whether MLD and control groups had similar intelligence scores or not.

Typically, MLD groups had worse working memory performance in at least some memory related variables than control groups. It turned out that the pattern of memory performance weakness depended on whether reading and/or intelligence was or was not similar across groups. On the one hand, when MLD children had poorer reading performance than control children then the MLD children showed substantially worse visuo-spatial working memory performance than control participants. In contrast, the discrepancy between MLD children and control children in verbal memory tasks was not as pronounced as on visual memory tasks. On the other hand, when the MLD and control groups had similar reading performance scores the pattern of results was the opposite: The MLD children showed much weaker verbal working memory performance than control participants but their visuo-spatial working memory performance differed relatively less from control participants. When both reading and intelligence measures were considered at the same time the results had the same pattern and they were even slightly more expressed.

The above outcomes suggest that there are at least two different types of MLD: In one type MLD appears with associated reading deficits. In this group of children we can also detect verbal short-term and working memory weakness and to a lesser extent visual short-term and working memory weakness. In another type of MLD, reading ability and intelligence is at normal level (note that the MLD children in [22] were in this category). In these children the primary cognitive signature is visuo-spatial short-term and working memory weakness and also a lesser extent they also have some verbal memory weakness. The results are consistent with various studies examining verbal memory function in low reading ability samples. These studies have shown that low readers have verbal STM and verbal WM deficits [29-30].

Overall, the above data are reminiscent of a double dissociation between two different types of MLD: One with associated reading problems and one without associated reading problems [21-22]. However, it is important to note that the data does not represent a full dissociation. This is because there is some visual memory weakness in the MLD children with reading problems and there is some verbal memory weakness in the MLD children with no reading problems. This 'subtype-independent memory weakness' may be the consequence of at least two factors. First, it is possible that there is a 'baseline' memory impairment in both subtypes of MLD, perhaps because some parts of short-term and working memory function draw on the same processes that are weak in both types of children. Second, the baseline memory weakness may be the consequence of having children with different profiles in groups. Such children may be borderline from the point of view that they may have slight reading problems but not strong enough problems so that they would be excluded from matched experimental groups. Including such borderline children could affect group level data.

MLD and working memory and the intraparietal sulcus of the human brain

A frequent observation is that children with MLD show abnormal structure and function in the intra-parietal cortex (IPS). This abnormality is often explained by assuming that an approximate magnitude representation residing in the IPS is impaired in MLD. However, a link between a putative deficient magnitude representation and altered IPS activity in MLD children is far from certain [22]. In fact, IPS abnormality can plausibly also be interpreted as a marker of altered working memory function in MLD.

First, the IPS is involved in various cognitive functions frequently implicated in numerical tasks, like working memory [31-36], attention [34; 37-39], inhibitory function [40-41] and spatial processing [42] and the proposed number sense [43]. Therefore, weaknesses in any of these functions could plausibly explain IPS abnormalities in MLD [22].

Second, as described above, several of the IPS related cognitive functions may actually contribute to verbal and visual working memory performance in the form of executive functions and/or as subcomponents of visual-spatial processes. Therefore, IPS related dysfunction in MLD may plausibly be related to a broad domain of working memory related cognitive processes rather than to a dysfunction or underperformance of a magnitude representation [43]. In fact, an overall view of the literature seems to suggest that there is more evidence for a working memory related explanation of IPS abnormalities in MLD than for a magnitude representation-based explanation [28; 44].

Educational implications

It is important to consider what tests were used to diagnose MLD. A practical definition of MLD is that students perform persistently weakly on mathematics but perform adequately on unrelated academic disciplines. MLD should not be due to low motivation to learn or to poor teaching methods. Depending on diagnostic tests and their score criteria there may be substantial variation in which students are diagnosed to have MLD. Boys and girls are equally likely to have MLD. It is not yet clear what exact subtypes of MLD exist, but it is likely that some forms of MLD are linked to reading problems (e.g. dyslexia) while other forms are not. The weakness of visual and verbal memory has often been linked to MLD. However, simple memory training is unlikely to improve MLD. As of today it is unclear what cognitive interventions can improve MLD.

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