How does the child's brain process numerical magnitude? Implications for learning mathematics

Magnitude is a core dimension of the semantics of number. How does a child's sense of magnitude develop and what does this mean for teaching mathematics to young children?

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Executive summary

- Numerical effects are elicited when children and adults compare numerical magnitudes. They are behavioral "signatures" of the brain networks responsible for magnitude processing.
- The intraparietal sulcus (IPS) is a key structure in the child's brain dedicated to processing numerical magnitudes. However, in younger children, other brain structures associated with memory, attention, and finger representation can help the IPS to deal with numerical information.
- For grasping the number concept, it is necessary to manipulate numerical quantities. This capacity is closely involved with learning mathematics. In fact, magnitude processing is an important skill associated with mathematical competence throughout life.
- Educators should present children with new mathematical challenges for "capitalizing on the sense of number" whenever possible. A curriculum based on this principle could strengthen the learning of mathematics in younger and older students.

Introduction

Magnitude is a core dimension of the semantics of number. From birth, we are continually comparing the numerical magnitudes of things in our life. Which one of two boxes has more chocolates? Is the place where I work more distant from my house than the park where I jog? Did I receive the correct amount of change in the market? There are innumerable examples. Numerical magnitude refers to a cardinal aspect of numbers, that is, when they denote the "numerosity" of a set (e.g., Butterworth, 1999). To grasp the magnitude concept, we need to learn the distinction between the transformations that do or do not modify the cardinality of a set (e.g., adding or removing objects in a set modify the cardinal; spreading or grouping the objects do not). We also need to be able to compare the numerosity of different sets (e.g., set A could be smaller, larger, or equal to set B). We first learn these principles through our experience with sets of real objects. Later on, we learn with symbols, such as Arabic numbers or words. The nature of magnitude representations in typical and atypical development have been commonly explored with magnitude comparison tasks using nonsymbolic (dots) or symbolic (digits) stimuli.

How the numerical magnitudes are processes: The tip of the iceberg

Numerical effects are elicited when children and adults compare numerical magnitudes. They are behavioral "signatures" of the brain networks responsible for magnitude processing.

When children and adults compare two numerical magnitudes, some effects are elicited in a typical manner. Interestingly, these effects are observable, robust, and constitute "signatures" of the status of the brain networks that specialize in numerical information. Using the iceberg metaphor, these are like "the tip of the iceberg," whereas the neural networks responsible for these effects are like "the submerged part." In this sense, we can obtain information directly and easily using the tip part, and indirectly using the part under the sea. These numerical effects have been replicated in many different languages, cultures, and numerical notations.

One of the most studied numerical effects is the *numerical distance effect* (see Figure 1). In simple words, the time needed to determine which is the larger (or smaller) quantity decreases with increasing numerical distance between two numerical quantities (e.g., people are faster at 2 versus 9 than 2 versus 3). Additionally, for an equal numerical distance, the time needed to compare quantities increases with the size of the quantities (e.g., people are faster at 2 versus 3 than 7 versus 8). This is called the *numerical size effect* and has also been reported many times in adults and children. On the other hand, the *semantic congruity effect* is also elicited when people make any type of magnitude comparison (e.g., "Which is smaller?"). That is, small values are more rapidly compared when participants are asked, "Which is smaller?," whereas large values are more rapidly compared when they are asked, "Which is larger?" In other words, people are faster to compare two numerical values when their overall magnitude is congruent with the semantics of the verbal phrasing of the question.

What happens in the child's brain when numerical magnitudes are processed?

The intraparietal sulcus (IPS) is a key structure of the child´s brain dedicated to processing numerical magnitudes. However, in younger *children, other brain structures associated with memory, attention, and finger representation can help the IPS deal with numerical information.*

There is now extensive evidence that a brain structure that runs horizontally at the middle of the parietal lobes, named the intraparietal sulcus, is very responsible for representing numerical magnitude (see Figure 2). Interestingly, in the child's brain, the activations generated when comparing numbers in Arabic format are situated in the core of the IPS while neural activations for comparing sets of dots are found very near to the IPS, but extending to neighboring structures which are related to the representation of fingers. So, this pattern of brain activations elicited by dots may reflect a link between fingers and nonsymbolic numerical processing. Actually, it has been reported that nonsymbolic numerical stimuli are more likely to elicit finger-based solution strategies than symbolic ones (Kaufmann et al., 2008; Butterworth, 2005; Gracia-Bafalluy and Noël, 2008).

On the other hand, there is evidence demonstrating the existence of age-related shifts in brain functional activity from frontal (anterior) regions to parietal (posterior) regions in response to number magnitude processing. In contrast to an adult's brain, the child's brain relies much more on frontal brain regions when numerical magnitudes are processed (Ansari and Dhital, 2006; Cantlon et al. 2006; Kaufmann et al., 2008; Rivera et al., 2005). The differences between children and adults probably reflect children's greater enrollment of areas involved in attention and working memory (frontal regions) for successfully processing numerical magnitudes. Such areas are presumably activated to compensate for "immature" representations of numbers in the IPS. Progressively, children improve their knowledge about numerical magnitude, and they need to rely less on these supplementary processes. Unfortunately, to date, developmental studies have focused on comparing children with adults. However, in the future, a more precise picture of the neural bases of the numerical development will be obtained from studies contrasting children of different ages.

Numerical magnitude processing and math competence

For grasping the number concept, it is necessary to manipulate numerical quantities. This capacity is closely involved with learning mathematics. In fact, magnitude processing is an important skill associated with mathematical competence throughout life.

Children's performance when they are asked to compare two digits is strongly associated with concurrent (Holloway and Ansari, 2009; Sasanguie et al., 2012; Reigosa-Crespo et al., 2012) and prospective maths competence (De Smedt et al., 2009; Sasanguie et al., 2013). This association has been systematically reported in younger and older children. However, research has provided mixed evidence concerning the relationship between performance on nonsymbolic comparison tasks and mathematical competence. Many researchers argue that these divergent findings could stem from the diversity of the tasks used (e.g., sometimes nonnumerical parameters such as density of the arrays, the areas of individual dots, their luminance, etc., are controlled, but sometimes not).

Surprisingly, the association between magnitude processing and mathematical competence is stable over the lifespan (Schneider et al., 2016). Alternatively, it might be that this association is mainly driven by an effect of numerical magnitude processing on early mathematical development, which then cascades into future math development throughout life. Or, as the second hypothesis, there might also be a causal influence in the opposite direction, so that gains in mathematical achievement cause improvements in magnitude comparison skills.

Implications for teaching maths

Educators should present children with new mathematical challenges for "capitalizing on the sense of number" whenever possible. A curriculum based on this principle could help strengthen the learning of mathematics in younger and older students.

Nowadays, there is robust evidence that specific brain networks we possess from birth are responsible for the capacity to manipulate numerical magnitudes. These findings could be a "usable knowledge" derived from neuroscience research with some implications for teaching mathematics. In this sense, educational neuroscientists strongly recommend, as a first step for teaching math in any grade, that teachers should "capitalize on this intuition" instead of immediately introducing abstract knowledge. Additionally, in educational settings, it is crucial to build the foundational number concepts first for strengthening the meaningfulness of numbers, especially the links between math facts and their component meanings.

Numerous attempts have been made to design educational interventions to foster the development of numerical magnitude processing. In the majority, the results have shown improvement in magnitude processing using nonsymbolic (Booth & Siegler, 2008; Hyde, Khanum & Spelke, 2014) and symbolic quantities (Obersteiner, Reiss & Ufer, 2013; Ramani & Siegler, 2011) with transfer to improvements in math competence.

The “numerical hour” in most kindergarten curricula comprises a wide variety of numerical activities, including number recognition, counting, comparing sets, playing board games, etc. These have been shown to have significant effects on children’s understanding of numbers and tests of early numeracy when they enter formal schooling. From these interventions, however, it is not possible to focus on stimulating numerical magnitude processing.

So, more relevant learning tasks could be those focused on very specific aspects of numerical magnitude processing which take into account the relevance of attentional and memory processes and the use of fingers in younger children. These kinds of tasks have been presented in game-like formats using both symbolic and nonsymbolic stimuli, and have been shown to have positive effects on children’s numerical magnitude processing. Some examples of activities that foster numerical magnitude processing are *Number Board Games* (Siegler, 2009), *Number Race Game* (Wilson et al., 2006), *Rescue Calcularis* (Kucian et al., 2011), and *The Estimator* (Vilette et al., 2010).

Finally, it is worth noting that most of these tasks have been used in kindergartners or with children from low-income backgrounds at risk of arithmetic problems who are especially impressionable by these interventions. Yet surprisingly, there is not an emphasis on numerical magnitude problems in the maths curriculum for older children. This omission produces deleterious consequences in learning maths. For example, fractions are often taught using the idea that fractions represent a part of a whole. This interpretation is important, but it fails to convey the essential information that fractions are numbers with magnitudes. One effective way to ensure that students understand this is to use number lines during instruction because they illustrate that each fraction corresponds to a given magnitude.

Key points for maths curriculum

- Focus on “number intuition” when new mathematical knowledge is presented, whenever possible.
- Focus on specific aspects of numerical magnitude processing (e.g., comparison tasks using varying levels of numerical distance and tasks for cementing links between nonverbal quantity representation and other developing symbolic representations of numbers, such as Arabic numerals or number words).
- Design numerical tasks using game-like formats. Take advantage of Information and Communication Technologies (ICT) tools.
- Include numerical magnitude problems for older children.

Glossary

Brain anterior regions – Here is mainly referred to as the frontal lobe. Located at the front of the brain, this is one of the four major lobes of the cerebral cortex in the mammalian brain. The frontal lobe is located at the front of each cerebral hemisphere and positioned in front of the parietal lobe and above and in front of the temporal lobe.

Brain posterior regions – Here is mainly referred to as the parietal lobe. This is one of the four major lobes of the cerebral cortex in the brain of mammals. The parietal lobe is positioned above the occipital lobe and behind the frontal lobe.

Cardinality – A property of sets indicating the number of elements in the set.

Finger-based strategies – Finger counting provides multisensory input, which conveys both cardinal and ordinal aspects of numbers. Recent data indicate that children with good finger-based numerical representations show better arithmetic skills and that training finger gnosis, or “finger sense,” enhances mathematical skills.

Nonnumerical quantities – Variables such as luminance and physical size or angles and lines offer nonnumerical quantity information by visual inspection of the stimuli. Based on this information we can judge the amount of nonnumerical stimuli (e.g., choosing the larger in size of “bikini” and “coat”).

References

Ansari, D. & Dhital, B. (2006). Age-related changes in the activation of the intraparietal sulcus during nonsymbolic magnitude processing: An event-related functional magnetic resonance imaging study. *Journal of Cognitive Neuroscience, 18*, 1820-1828.

Booth, J. L. & Siegler, R. S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development, 79,* 1016–1031. doi:10.1111/j.1467-8624.2008.01173.x

Butterworth, B. (1999). *The Mathematical Brain.* Macmillan.

Butterworth, B. (2005). The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry, 1*, 3-18.

Cantlon, J. F., Brannon, E. M., Carter, E. J. & Pelphrey, K. A. (2006). Functional imaging of numerical processing in adults and 4-yr-old children. *PLoS Biology, 4*, e125.

Gracia-Bafalluy, M. & Noël, M. P. (2008). Does finger training increase young children's numerical performance? *Cortex, 44*, 368-375.

Holloway, I. D. & Ansari, D. (2009). Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children's mathematics achievement. *Journal of Experimental Child Psychology, 103*, 17-29.

Hyde, D.C., Khanum, S., & Spelke, E.S. (2014). Brief nonsymbolic, approximate number practice enhances subsequent exact symbolic arithmetic in children. *Cognition, 131*, 92– 107. doi:10.1016/j.cognition.2013.12.007

Kaufmann, L., Vogel, S. E., Wood, G., Kremser, C., Schocke, M., Zimmerhackl, L. B. & Koten, J. W. (2008). A developmental fMRI study of nonsymbolic numerical and spatial processing. *Cortex, 44*, 376-385.

Kucian, K., Loenneker, T., Martin, E. & von Aster, M. (2011). Non-symbolic distance effect in children with and without developmental dyscalculia: a parametric fMRI study. *Developmental Neuropsychology*, *36*, 741-62.

Obersteiner, A., Reiss, K., & Ufer, S. (2013). How training on exact or approximate mental representations of number can enhance first-grade students' basic number processing and arithmetic skills. *Learning and Instruction, 23*, 125–135. doi:10.1016/j.learninstruc.2012.08.004

Ramani, G.B., & Siegler, R.S. (2011). Reducing the gap in numerical knowledge between low- and middle-income preschoolers. *Journal of Applied Developmental Psychology, 32*, 146–159. doi:10.1016/j.appdev.2011.02.005

Reigosa-Crespo, V., Valdés-Sosa, M., Butterworth, B., Estévez, N., Rodríguez, M., Santos, E., Torres, P., Suárez, R. & Lage, A. (2012). Basic numerical capacities and prevalence of developmental dyscalculia: The Havana survey. *Developmental Psychology, 48*, 123-135.

Rivera, S. M., Reiss, A. L., Eckert, M. A. & Menon, V. (2005). Developmental changes in mental arithmetic: Evidence for increased functional specialization in the left inferior parietal cortex. *Cerebral Cortex*, 15, 1779-1790.

Sasanguie, D., De Smedt, B., Defever, E. & Reynvoet, B. (2012). Association between basic numerical abilities and mathematics achievement. *British Journal of Developmental Psychology, 30*, 344-357.

Sasanguie, D., Göbel, S. M., Moll, K., Smets, K. & Reynvoet, B. (2013). Approximate number sense, symbolic number processing, or number-space mappings: what underlies mathematics achievement? *Journal of Experimental Child Psychology, 114*, 418-431.