How does the brain learn simple arithmetic? The impact of teaching methods

A fundamental goal of elementary education is to achieve fluency with simple (i.e., single-digit) arithmetic. That is, by the end of 5th grade children are expected to quickly and effortlessly respond 5 when faced with 2 + 3, or 24 when faced with 6 × 4. How do they achieve this?

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Executive summary

- Mastering basic arithmetic facts such as 2 × 3 or 3 + 4 is a requirement for more advanced mathematical skills and a primary goal of elementary education.

- Poor knowledge of arithmetic facts is a hallmark of math learning difficulties in children.

- Neuroscience research suggests that learning facts by repeated calculation can be just as efficient as rote learning, at least for some operations.

Introduction

A fundamental goal of elementary education is to achieve fluency with simple (i.e., single-digit) arithmetic. That is, by the end of 5th grade children are expected to quickly and effortlessly respond 5 when faced with 2 + 3, or 24 when faced with 6 × 4. This is not only because knowledge of such *arithmetic facts* are important in everyday life, but also because basic arithmetic skills in children are thought to provide a scaffold upon which more advanced mathematical skills are built. For instance, basic arithmetic skills are known to correlate with later math skills, and brain activity during arithmetic calculation in high school seniors is associated with their math scores[1]. Difficulties with acquiring simple arithmetic facts are also a hallmark of mathematical learning disability, which affects around 5% of children worldwide[2]. Therefore, there is a general consensus that all children need to master basic arithmetic facts by the end of elementary school, before moving on to more advanced aspects of math.

How are arithmetic facts learned? Learning by rote versus learning by doing

Broadly speaking, there are two main ways to learn simple arithmetic facts. The first one is rote memorization. That is, children can be taught associations between a given number combination (e.g., 2 + 3) and its answer (e.g., 5). A defining feature of rote memorization is that it does not require an understanding of the relationship between a number combination and its associated result. Rather, it relies on *declarative memory*, a type of long-term memory that supports the recollection of facts and events (such as, what is the capital of France? or who is the president of the United States?). In the domain of arithmetic learning, rote memorization is often associated with tables in which number combinations are organized (see Figure 1).
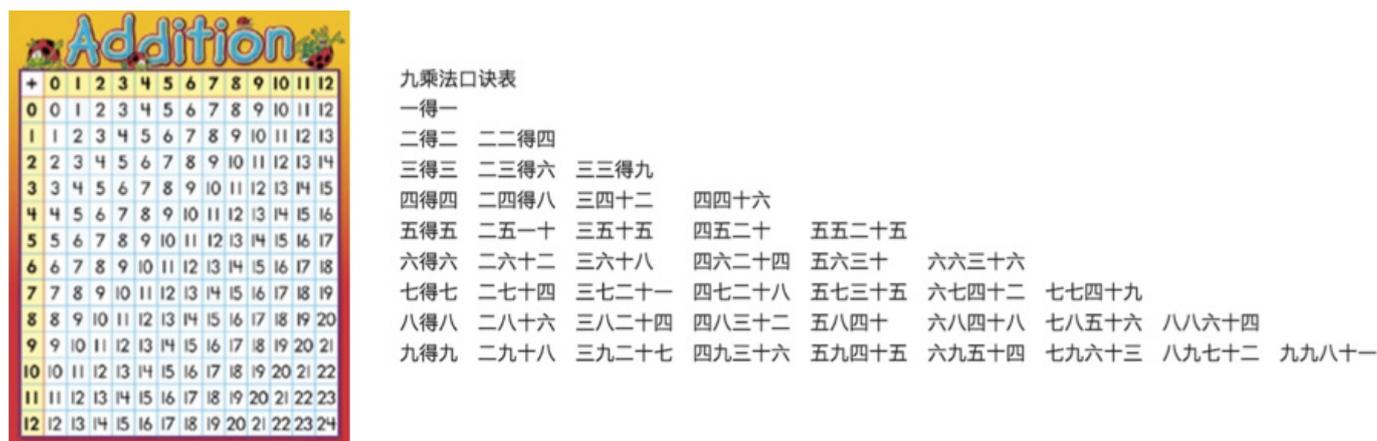

Figure 1. Examples of a Western addition table (left) and of a Chinese multiplication table, also called nine nine song (right).

Another way to learn arithmetic facts is to repeatedly practice them until mastery is achieved. This, of course, requires an understanding of the fundamental principles and relations underlying arithmetic operations. It also involves the use of increasingly efficient calculation strategies over the course of learning. Consider, for example, the way children often learn addition. Relying on external aids such as fingers or objects, young children may first use what is called the *counting-all* procedure: They count out two sets of objects before combining them and counting the newly formed set. For instance, adding two to three would require first counting out two objects, then counting out three objects, and finally putting them together in order to count out the formed set (i.e., "one, two, three, four, five"). Older children may then realize that objects can

be replaced by counting words (e.g., "two" and "three") and that they do not necessarily need to use external aids such as fingers when adding numbers. This realization is often accompanied by the appearance of the more sophisticated *counting-on* procedure, according to which children start to count from one of the two number words. For instance, when adding two to three, children may start from "three" and count "four, five." Alternatively, they may start from "two" and count "three, four, five." In using this *counting-on* procedure, many children will also realize that starting from the largest number is more efficient than starting from the smaller number. Overall, then, arithmetic learning is often characterized by the use of increasingly efficient and sophisticated procedures over development.

No matter how efficient calculation procedures may become, however, it is often thought that the end product of arithmetic learning ought to be direct recollection of arithmetic facts from memory. That is, even when no rote memorization is explicitly used, the repeated practice of calculation procedures (or "learning by doing") would lead to an association between number combinations and their answers in declarative memory. For example, 2 + 3 might be automatically associated with 5 in memory after children repeatedly come up with five when adding two to three. Yet, recent neuroscience findings point to a quite different story. In fact, the way simple arithmetic is taught in school might have a significant impact on the way educated adults process simple arithmetic facts.

The impact of learning methods on the arithmetic brain

Over the past two decades, a number of brain imaging studies have investigated the way arithmetic facts are processed in the educated brain. Strikingly, although adults might unequivocally report having memorized the answer of 3 + 2, 3 × 2, or 3 − 2, the brain appears to process these problems using quite different brain regions. For example, processing simple multiplication facts such as 3 × 2 has been shown to rely on areas that support verbal memory in the left side of the brain[3] (see brain regions in green in Figure 2). This is consistent with the aforementioned view that associations between numerical combinations and multiplicative answers are indeed stored in long-term memory, most likely in the form of verbal routines ("two times three equals six"). However, when adults solve simple addition and subtraction facts, they appear to engage other areas located at the back of the brain (see brain regions in red in Figure 2). Quite surprisingly, these brain regions are very similar to the regions that are thought to be engaged when younger children explicitly manipulate numerical quantities[3,4]. How is that possible? Part of the answer might lie in the fact that not all arithmetic facts are learned the same way in school. For example, while single-digit multiplication facts are most often drilled (e.g., in multiplication tables, see Figure 1), this is not usually the case for single-digit addition and subtraction facts. Therefore, although it is often assumed that facts that are repeatedly practiced end up being retrieved from memory (in the same way as drilled facts are), this might not necessarily be the case for all facts.
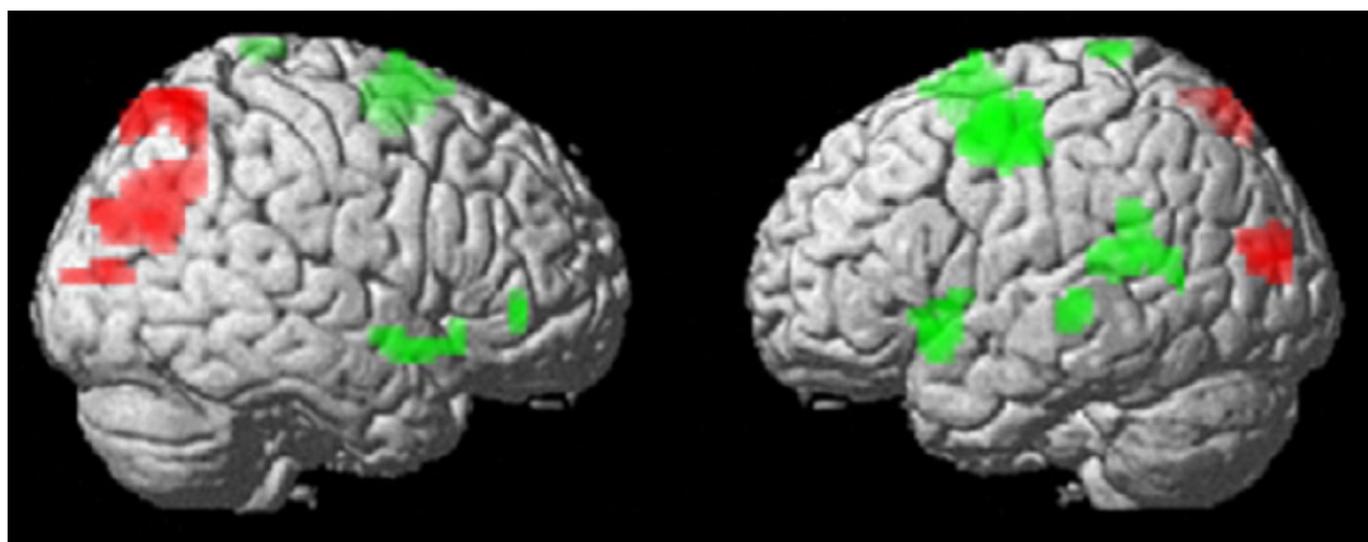

Figure 2. The arithmetic brain. Regions activated when adults process multiplication facts (green) and addition facts (red) on a 3D representation of the brain taken using a magnetic resonance imaging scanner (Reproduced from ref. [4]).

That hypothesis has been put to the test by Delazer and colleagues[5] in a brain imaging study. The researchers recruited adult participants and trained them over a week on two novel complex arithmetic operations, symbolized by the artificial signs § (9 § 15 = ?) and # (4 # 17 = ?). Participants learned the answers of "§" problems by repeatedly applying to these problems the strategy "subtract the second element from the first element, add one, and add the second element" (e.g., *[(15 −*

9) + 1] + 15 = 22 for the problem *9 § 15 = ?*). In contrast, participants learned the answers of "#" problems simply by memorizing them (e.g., 4 # 17 = 28). At the end of the training, both types of problems were mastered by participants and solved with roughly similar response times. Yet, an examination of their brain activity indicated that the brain regions processing the operations "§" and "#" were not the same. Whereas solving the "#" operation (learned by drill) mostly activated regions associated with verbal memory (i.e., the brain regions in green in Figure 2), solving the "§" operation (learned by repeated calculation) mostly activated regions known to be involved in numerical processing (i.e., the brain regions in red in Figure 2). This led the researchers to propose that learning facts by repeatedly calculating them might "lead to faster and more efficient strategies," rather than to a recollection of the answer from memory[6].

Of course, the experiment described above is somewhat artificial because it attempts to "simulate" learning in adults who are already proficient with arithmetic. But brain imaging research in children also shows a similar dissociation between facts that are learned by rote and facts that are learned through repeated calculation. For example, Prado and colleagues[7] showed that as children become proficient with arithmetic, they increasingly recruit brain regions involved in verbal memory when processing single-digit multiplication facts (typically learned by rote). However, increased proficiency with single-digit subtraction (an operation that is typically learned by repeated calculation) is associated with increased reliance on brain regions that underlie numerical processing. Thus, although we might be convinced—as adults—to know the results of 3 + 2 or 3 – 2 by heart, it is possible that our brain does not simply retrieve these results from memory. Instead, our brain might reconstruct these results by using calculation procedures that may not be that different from the counting procedures used by young children, only much more efficient and automatic. This idea is supported by several recent studies that closely examined the time adults take to solve simple addition problems involving numbers from 1 to 4[8,9]. Educated adults overwhelmingly report knowing answers of these problems by heart. Yet, researchers find that the time it takes to solve these problems increases linearly with the size of the numbers. In other words, adults take 20 milliseconds (thousandths of a second) longer to solve 1 + 2 than 1 + 1, 20 milliseconds longer to solve 1 + 3 than 1 + 2, and 20 milliseconds longer to solve 1 + 4 than 1 + 3. It is quite difficult to explain this pattern if we assume that the answers of these problems are simply retrieved from memory. Instead, this linear increase in solution times suggests one thing: Adults might still count, even when solving these very basic and well-practiced problems. Of course, such counting may be unconscious and "automatized," and therefore much faster and effective than in children. For instance, it could take the form of a rapid scanning of a sequence of numbers[8,9]. This would explain why it takes a little longer to go from 1 to 4 (when calculating 1 + 3) than to go from 1 to 3 (when calculating 1 + 2). Overall, cognitive neuroscience research tells us that the way arithmetic facts are learned in school may have a tremendous impact on the way these facts are processed later on in the adult brain.

Conclusion

In sum, neuroscience research indicates that there are multiple ways children can reach arithmetic fluency. It is sometimes claimed that rote learning may be the most efficient method for acquiring arithmetic facts because it is more likely to lead to direct retrieval from memory (and direct retrieval is often thought to be the most efficient way to process simple arithmetic problems). Yet, neuroscience studies suggest that procedural learning can be just as efficient because it could lead to an automatization of calculation procedures. For instance, the practice of counting when learning to add numbers might "train" the brain mechanisms supporting numerical manipulation so much that they end up operating automatically and unconsciously in adults. Of course, procedures might not be efficient for all operations and rote learning may still have an important role in several aspects of arithmetic learning (e.g., single-digit multiplication). But neuroscience research suggests that the brain has several ways to efficiently process arithmetic facts, and learning by doing should be promoted at least as much as learning by rote.

References

1. Price, G.R., M.M. Mazzocco, and D. Ansari, Why mental arithmetic counts: brain activation during single digit arithmetic predicts high school math scores. *J Neurosci*, 2013. 33(1): p. 156-63.

2. Kaufmann, L., et al., Dyscalculia from a developmental and differential perspective. *Frontiers in Psychology*, 2013. 4: p. 516.

3. Prado, J., et al., Distinct representations of subtraction and multiplication in the neural systems for numerosity and language. *Hum Brain Mapp*, 2011. 32(11): p. 1932-47.

4. Zhou, X., et al., Dissociated brain organization for single-digit addition and multiplication. *Neuroimage*, 2007. 35(2): p. 871-80.

5. Delazer, M., et al., Learning by strategies and learning by drill–evidence from an fMRI study. *Neuroimage*, 2005. 25(3): p. 838-49.

6. Ischebeck, A., et al., How specifically do we learn? Imaging the learning of multiplication and subtraction. *Neuroimage*, 2006. 30(4): p. 1365-75.

7. Prado, J., R. Mutreja, and J.R. Booth, Developmental dissociation in the neural responses to simple multiplication and subtraction problems. *Dev Sci*, 2014. 17(4): p. 537-52.

8. Barrouillet, P. and C. Thevenot, On the problem-size effect in small additions: Can we really discard any counting-based account? *Cognition*, 2013. 128(1): p. 35-44.

9. Uittenhove, K., C. Thevenot, and P. Barrouillet, Fast automated counting procedures in addition problem solving: When are they used and why are they mistaken for retrieval? *Cognition*, 2015. 146: p. 289-303.